Journal of Convex Analysis
Volume 30 (2023), No. 1, 401–402

Corrigendum to “On Numerical Radius and Crawford Number Attainment Sets of a Bounded Linear Operator”

Debmalya Sain
Dept. of Mathematics, Indian Inst. of Information Technology, Raichur, Karnataka, India
saindebmalya@gmail.com

Arpita Mal, Pintu Bhunia
Dept. of Mathematics, Indian Institute of Science, Bengaluru, Karnataka, India
arpitamalju@gmail.com, pintubhunia5206@gmail.com

Kallol Paul
Dept. of Mathematics, Jadavpur University, Kolkata, West Bengal, India
kalloldada@gmail.com

Received: February 8, 2023
Accepted: February 9, 2023

The purpose of this article is to present correct versions of Theorems 2.4 and 2.8 of a previous article of the authors [*On numerical radius and Crawford number attainment sets of a bounded linear operator*, J. Convex Analysis 28 (2021) 67–80].

Keywords: Numerical radius, Crawford number, norm attainment set, bounded linear operator.

2010 Mathematics Subject Classification: Primary 47A12; Secondary 46B20.

In the original article, Theorem 2.4 is stated incorrectly. The following theorem is a correct version of [1, Th. 2.4].

Theorem 1. *Let $\mathbb{H}$ be a complex Hilbert space. Let $T \in \mathbb{L}(\mathbb{H})$ be such that $c(T) > 0$. Suppose $x \in c_T$ and $y \in x^{\perp} \cap S_{\mathbb{H}}$. Then $|\langle Tx, y\rangle| = |\langle Ty, x\rangle|$. In particular, if $\Re(\langle Tx, x\rangle) = 0$, then $\langle Tx, y\rangle = \langle x, Ty\rangle$ and if $\Im(\langle Tx, x\rangle) = 0$, then $\langle Tx, y\rangle = -\langle x, Ty\rangle$.*

Proof. Suppose that $\langle Tx, x\rangle = a + ib, \langle Tx, y\rangle = \alpha + i\beta, \langle x, Ty\rangle = \gamma + i\delta$. Without loss of generality, assume that $c(T) = 1$. From [1, Th. 2.2], it follows that x is an eigenvector of $\langle\Re(T)x, x\rangle\Re(T) + \langle\Im(T)x, x\rangle\Im(T)$ corresponding to the eigenvalue $c(T) = 1$. Therefore,

$$\begin{aligned}
\langle\Re(T)x, x\rangle\Re(T)x + \langle\Im(T)x, x\rangle\Im(T)x &= x\\
\Rightarrow \langle\Re(T)x, x\rangle\langle\Re(T)x, y\rangle + \langle\Im(T)x, x\rangle\langle\Im(T)x, y\rangle &= 0\\
\Rightarrow \langle Tx + T^*x, x\rangle\langle Tx + T^*x, y\rangle - \langle Tx - T^*x, x\rangle\langle Tx - T^*x, y\rangle &= 0\\
\Rightarrow \Re(\langle Tx, x\rangle)\{\langle Tx, y\rangle + \langle x, Ty\rangle\} - i\Im(\langle Tx, x\rangle)\{\langle Tx, y\rangle - \langle x, Ty\rangle\} &= 0\\
\Rightarrow a\{\alpha + \gamma + i(\beta + \delta)\} - ib\{\alpha - \gamma + i(\beta - \delta)\} &= 0.
\end{aligned}$$

Thus, $$a(\alpha + \gamma) + b(\beta - \delta) = 0 \text{ and } a(\beta + \delta) - b(\alpha - \gamma) = 0. \tag{1}$$

ISSN 0944-6532 / \$ 2.50

Note that, if $b = 0$, then $a \neq 0$, which implies that $\alpha + \gamma = 0 = \beta + \delta$. Thus, in this case $\langle Tx, y\rangle + \langle x, Ty\rangle = 0$.

Similarly, if $a = 0$, then $b \neq 0$, which gives $\beta - \delta = 0 = \alpha - \gamma$. Thus, in this case $\langle Tx, y\rangle - \langle x, Ty\rangle = 0$.

Let $a \neq 0$ and $b \neq 0$. Then solving (1), we get $(\beta^2 - \delta^2 + \alpha^2 - \gamma^2) = 0$. Hence, $|\langle Tx, y\rangle| = |\langle Ty, x\rangle|$. This completes the proof. □

Remark 2. Theorem 1 does not hold if $c(T) = 0$. Consider $T = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. Then there exists $x \in c_T$ and $y \in x^\perp \cap S_{\mathbb{H}}$ such that $| \langle Tx, y\rangle | \neq | \langle Ty, x\rangle |$.

The next theorem is a correct version of [1, Th. 2.8].

Theorem 3. *Let $\mathbb{H}$ be a complex Hilbert space. Let $T \in \mathbb{L}((\mathbb{H})$ be a non-zero operator. Suppose $x \in M_{w(T)}$ and $y \in x^\perp \cap S_{\mathbb{H}}$. Then $|\langle Tx, y\rangle| = |\langle Ty, x\rangle|$. In particular, if $\Re(\langle Tx, x\rangle) = 0$, then $\langle Tx, y\rangle = \langle x, Ty\rangle$ and if $\Im(\langle Tx, x\rangle) = 0$, then $\langle Tx, y\rangle = -\langle x, Ty\rangle$*

Proof. Without loss of generality, assume that $w(T) = 1$. Assume $\langle \Re(T)x, x\rangle = \cos\theta$ and $\langle \Im(T)x, x\rangle = \sin\theta$ for some $\theta \in [0, 2\pi)$. From [1, Th. 2.6], it follows that x is an eigenvector of $\cos\theta\Re(T) + \sin\theta\Im(T)$ corresponding to the eigenvalue $w(T) = 1$. Therefore, $\cos\theta\Re(T)x + \sin\theta\Im(T)x = x$ and so

$$\frac{e^{-i\theta}T + e^{i\theta}T^*}{2}\, x = x.$$

Now consider $y \in x^\perp \cap S_{\mathbb{H}}$. Then

$$\left\langle \frac{e^{-i\theta}T + e^{i\theta}T^*}{2}\, x,\, y \right\rangle = 0$$

and so $\langle e^{-i\theta}Tx, y\rangle = -\langle e^{i\theta}T^*x, y\rangle$. Thus $|\langle Tx, y\rangle| = |\langle x, Ty\rangle|$. The remaining part of the proof follows trivially. □

References

[1] D. Sain, A. Mal, P. Bhunia and K. Paul: *On numerical radius and Crawford number attainment sets of a bounded linear operator*, J. Convex Analysis 28 (2021) 67–80.