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From Stochastic Programming to Variational Analysis

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This paper presents some thoughts that come into my mind when I look back on my scientific life.

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I decided to devote a few lines to how I ended up working in these fields and how life and colleagues provided guidance to some of my work. These few lines only concern my path to stochastic programming (for the deeper issues that need to be confronted in Variational Analysis, cf. the book on that subject with R.T. Rockafellar).

I come from a business family and I was supposed to follow in my grand- and father steps. They co-managed the family sawmills and associated services. My father introduced a number of innovative procedures that improved productivity. But he also had some interest in real estate (of limited scope) which during the last years of his live led him to act as the unofficial architect of a number of houses that he would rarely sell but rent out. As I was finishing high school, he co-bought a cardboard factory where I would manage the day-to-day administrative tasks and help expand its commercial reach. Not withstanding some success from time to time, during the 5.5 years I worked at "Cartonnerie d'Alsenberg", this expansion never took place and eventually my father sold his shares. Of course, in the process I had lost a real university education. I had been following an evening course that led to a degree in business administration but never got much interested in it except for the lectures dealing with law and the economics seminar. Now, I was liberated from the accounting and various administrative deadlines of the cardboard factory and free to choose a different path. I had been an excellent math-student in high school and with my friend, Morel de Westgaver, we had studied various mathematical domains to enliven our literature classes but at that point, these mathematical excursions were six years ago. Trying to catch up, I decided to follow a super-full collection of mathematics courses at Université Libre de Bruxelles and University of Louvain. I was particularly fascinated by Algebraic Structures and thought that would be included in my forthcoming activities. But this is not how it worked out. Visiting a book store, the owner pointed to a book that was applying mathematical techniques to deal with some business issues. It was an introduction to Operations Research. It seemed to fit the areas with which I already had some familiarity and the following semester I included in my schedule an Operation Research course. In fact, the only one offered in Belgium (at Louvain), taught by Professor Jacques Drèze (many times nominated for a Nobel Prize) who became a very informal advisor. Aware of my interest in optimization, Drèze suggested that I apply to University of California at

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Berkeley where Dr. George Dantzig would join its faculty the following year. This is exactly what I did not realizing that my esoteric background might not guarantee acceptance. My goal was to follow a program in Enginering Sciences for one or two semesters and then find a position that would support my nascent family. But when I described my plans to Dantzig, he suggested a clear alternative: finish your Ph.D. here and then you can go/do whatever you want, meanwhile you can be one of my Research Assistants. And this is exactly what I did being thankful to this day for his advice and support. At the beginning of my third year in Berkeley, Dantzig invited me to his house and while watering his roses, he informed me of my thesis subject: "Roger, you like optimization, probability and economics, you should be working in uncertainty"; which I even then understood as decision making under uncertainty, eventually also known as stochastic programming. I certainly wasn't familiar with his wording but decided to give it a try. In retrospect I am impressed by Dantzig capability to perceive my place in our scientific community as well as my limitations. Our relationship was never close but at some critical times he came with outstanding suggestions about executive-type decisions about myself as well as that of my research-group, students and asssociates. In Berkeley, I ended with three friends following more or less similar objectives as mine. Richard Cottle, Ellis Johnson and Richard Van Slyke became, inofficially, my technical advisors co-participating in a joint seminar that covered advanced/up-to-date topics in optimization; all of them eventually playing important roles in the mathematical programming community.

My initial literature search smoked out no more than 14 articles dealing with optimization under uncertainty. This was a good omen, suggesting there wouldn't be such a heavy load of earlier results to take into account, but I got disillusioned when I realized that 11 of these articles or pre-publications drafts included serious errors that invalidated their proclaimed results and I wondered if I should join such an impaired community. The couple of articles by Dantzig were solid and I figured that as long as I would build my work on his it would work out. In addition to a couple of outliers, the research community in stochastic programming was, in those days, divided in two major groupings both motivated directly or indirectly by application(s) that had initiated their research. One led by Professors Abraham Charnes and William Cooper called "Chance-Constrained" programming was based on applications coming from oil exploration. The other led by Dantzig, and a couple of his associates, was initiated, I suspect, by the search for an optimal delivery of supply/replacement parts, (at the end of WWII and its aftermath), to american and allied bases spread around the globe; this is not factual but simply an educated(?) guess. Eventually, one referred to this class of problems as "stochastic programs with recourse".

A stochastic program with chance-constraints, like all stochastic programs, accept the fact that some of its constraints were 'probabilistic', i.e., that the coefficients of these constraints are 'uncertain' and from a technical viewpoint could be considered random variables. A too brief formulation would read:

$$\min E\{c(x)\} \text{ such that } x \in D,\ \text{prob.}\{x \in D_u(\xi)\} \geq \alpha\,,$$

where $D_u(\cdot)$ is the set determined by the stochastic constraints and α is a coefficient selected to satisfy some reliability considerations. Altough the formulation of the problem is simple it's actually a sophisticated problem and even its basic properties are difficult to unearth and clarify; Professor Andras Prekopa work in this area has been remarkable, but, so far, difficult to exploit computationally.

In '64, I gave a lecture about my so-far results about stochastic programming with recourse at the International Congress on Mathematical Programming in London and Dantzig came to my lecture and as we left the room, he told me that I had more than enough for a thesis and should write it up and that's what I did. In many ways my Ph.D. thesis was a formalization of Dantzig's work which rendered the technique applicable to a wider class of problems. The following September I joined BSRL (the Boeing Scientific Research Lab), in Seattle, which allowed its members to pursue their own research program.

Clearly, this now left me only one major task: find an efficient procedure for solving stochastic programs with recourse. At the outset this didn't turn out that promising. A minor but unexpected result helped rekindle my hopes. Again, a simplified formulation of a linear version of the problem could read:

$$\begin{gathered}\text{find } x, \xi \mapsto y(\xi) \text{ that minimizes } c(x) + E\{q(y(\xi))\} \\ \text{such that } x \in D,\ x \in D_u(\xi; y(\xi)), \forall \xi\end{gathered}$$

where $y(\xi)$ is a recourse (decision) availabe when x fails to satisfy $x \in D_u(\xi, \cdot)$ for some realizations of ξ. I showed that irrespective of the distribution of the random variables, the set $K \subset D$ satisfying the probabilistic constraints was a polyhedral set. Consequently, there was some hope that one could rely on linear-programming type tools, including pivoting, to find a solution. In fact, Dr. Richard Van Slyke had sketched out such an algorithm based on a dual of the "Dantzig-Wolfe decomposition method". After formalizing his work, it turned out that the constraints of a stochastic program with recourse and those of the optimal control problem motivating Van Slyke both had a L-shaped structure which is the name under which the method became known and utilized. Variants of this method were proposed and implemeted successfully among others by Professors Julie Higle, Suvrajeet Sen, David Woodruff and Jean-Paul Watson, Jeffrey Linderoth and Michael Ferris.

So far we have been focused on computational issues in an environment where the analysis of the stability of the solutions does play a significant role since 'uncertainty' about the model itself must be taken into account. Moreover, one should also not bypass the possibility of approximating the given problem. The first result/progress in this direction came via a question about the polars of convex cones. The question can be summarized as follows: Does the proximity of two convex cones guarantee the same property for their polars? A proof of this did turn out to be relatively easy but my office-colleague, Dr. David Walkup, thought one could prove a much stronger property, namely that there was an isometry with repect to a well defined distance between convex cones and their polars. We came up with the proof and the distance in question turned out to be what is presently called the epi-distance and is used not just in this context. A couple of years earlier Professor R. Wijsman had actually shown that if a collection of convex functions f^n epi-converges to f then so did their conjugates; epi-convergence meaning that their epigraphs set-converge. One could imply the epi-convergence results for convex functions from that for polar cones but now we could include quantitative estimates for the convergence rate. In fact,as was made clear later, the epi-distance was the appropriate concept to handle continuity in the stochastic, as well as the deterministic, optimization framework.

In combination with the impressive subdifferentiability paradigm developed by my hiking friend, Professor R. T. Rockafellar, these continuity results completed the tools for a new Analysis. To confirm these results, a good test was to derive a duality result. This time, the problems had to be viewed as an infinite dimensional optimization problem. The same applied to the results for optimal control problems of the L-shaped type and I was very proud when I could show them to Rockafellar who immediately showed me a proposed article he had just sent a few days earlier for publication. In fact, the results were fundamentally identical except that his work was more complete by relying on (convex) conjugate functionals that enriched the connections to perturbations of the given (original) problem. This eventually led to our extensive collaboration which started on a short train trajectory from Cologne to Bonn where we both were invited to give some lectures (Rheinische Friedrich-Wilhelms-Universität Bonn). First on duality for stochastic programs and later extending to Variational Analysis that is able to deal with non-differentiable functions and (modestly) includes Newton Calculus. To elaborate on this does take at least a volume or two and will not be attempted here but of course is available in the literature, in particular in our book on Variational Analysis.

There are some collaborators, "influencers", friendly colleagues that provided important imput/comments at a later stage of my development and my story wouldn't be complete without mentioning them. They certainly would include: Hedy Attouch, Zvi Artstein, Gabriella Salinetti, David Woodruff, Jean-Pierre Aubin, David Blackwell and Jong-Shi Pang. But without the support of my wife Maria who provides me with many years of undisturbed very happy life, none of this would never have come to anything. She doesn't get credit for any specific equation or lemma which appears in my work although these results wouldn't exist without her generous, hidden, co-authorship.