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Looking back ...

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This paper gives an overview of my mathematical life, in particular to my relationship with Convex Analysis, Stochastic Optimization and the Calculus of Variations.

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I didn't know I wanted to be a mathematician until I somehow unexpectedly became one and found deep pleasure in research and discovery. That was toward the end of 1958 when I got introduced to the linear programming duality theorem and the simplex method, which I excitedly thought I could improve. As I write this now, 66 years later to the month, problem structure in optimization and the ways it might aid in the design of algorithms still grab my attention. There's always more to explore, more to write up and speak about, and that never-ending thrill of truth and beauty that mathematics delivers.

So much has happened on the way. As a student, I gradually shifted into higher courses in mathematics, because they attracted my curiosity and suited my nature, but I never imagined that as a career. The knowledge input was very broad. By the time I was writing my dissertation, I had learned about many things in advanced algebra, topology and complex analysis that hardly mattered later, as well as the real analysis and function space theory that did. A course in the mathematics behind quantum mechanics was keenly motivating to me. An electron could be viewed as an element of a Banach algebra of operators, for instance. There was no way to understand modern discoveries in physics without the development of new models beyond Newton. Maybe I could help by laying new foundations to support that in other subjects, such as economics?

In the years that followed, there was of course first the development of convex analysis that took a lot of my attention and connected me to Jean Jacques Moreau and other enthusiastic researchers. Having started in optimization of the kind centered on finite-dimensional problems and algorithms, I benefited greatly from that connection to France, where theory was welcome even if it didn't immediately solve something practical.

Maximal monotone operators emerged in the late 1960s as valuable in nonlinear functional analysis, and the insight that they are actually key objects in convex analysis was a landmark in my thinking. Here was an example of a concept from the upper reaches of infinite-dimensional theory, far from obvious or easy to picture, which could nevertheless also be important to numerical optimization through the proximal point algorithm.

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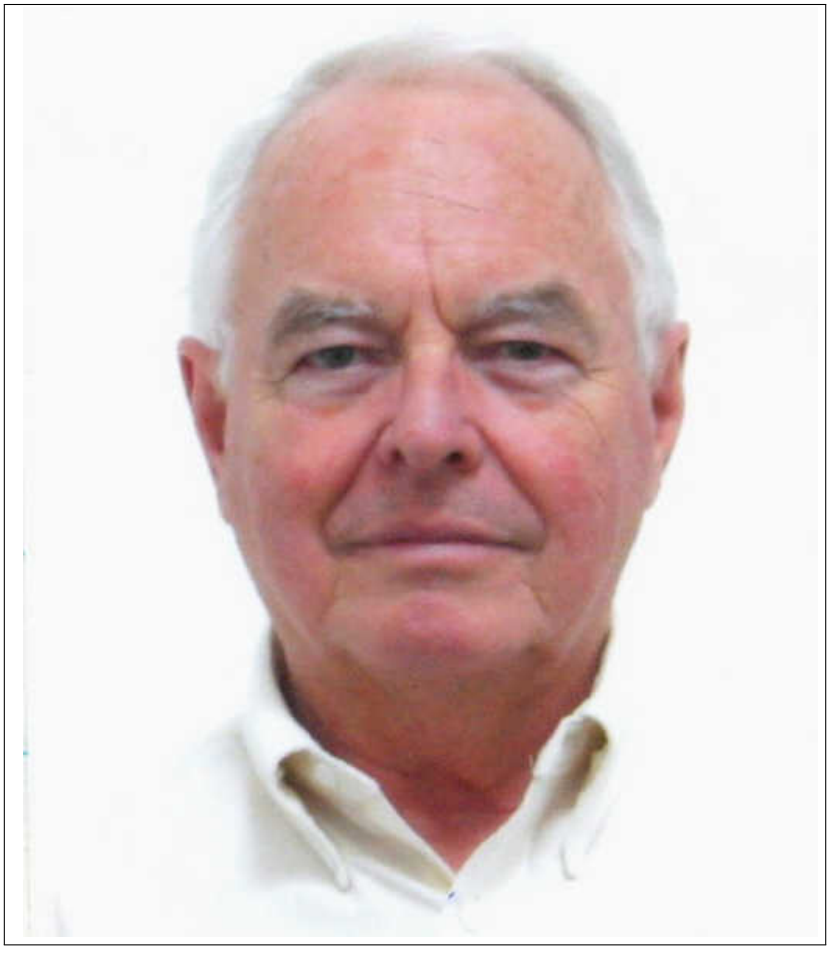

Ralph Tyrrell Rockafellar

Powerful applications revolve around augmented Lagrangians and problem decomposition techniques, such as the progressive hedging algorithm in stochastic programming, and splitting methods that are at the forefront now in machine learning. To this very day, I continue to work avidly on such methodology, especially its extension to nonconvex optimization through localized monotonicity as progressive decoupling.

In the Soviet Union back then, optimization was dominated by control, but I was more interested in its traditional counterpart, the calculus of variations — which I had learned as a part of that introduction to quantum mechanics. It struck me that the classical problems of Lagrange and Bolza could be recast by allowing the integrands to be closed proper convex functions, rather than smooth functions. That would result in convex optimization problems which could even be dualized. This idea so captivated me that I spent many years on it, working with partners to develop generalized Euler-Lagrange equations, Hamiltonian equations and transversality conditions that in fact could even turn out to cover the maximum principle in optimal control.

That project required more than convex analysis. It was perhaps the main reason why I believed it was so important to create a theory of variational analysis which could robustly extend beyond convexity while taking advantage of some of its groundbreaking features like epigraphs and subgradients. Years went into that, but other projects continued in parallel. There were all the developments in stochastic programming. And so much on metric regularity, variational inequalities and implicit solution mappings. Then, starting in 1999, I got into risk theory in finance and reliable engineering design, so different, yet it led to a couple of my most cited papers. Why that subject? It's another domain of exercise for convex analysis and duality. And it promotes better modeling in stochastic optimization and machine learning through hidden ties to statistics and alternative approaches to linear regression and estimation.

Through all of this, although I did many things on my own, lots of efforts were long-term collaborations with students or colleagues in other places, above all with Roger Wets and Asen Dontchev and our jointly authored books, and Stan Uryasev for everything on risk. There is so much in all of those collaborations to be grateful for, and not just the mathematics.

I'm lucky to have lived long enough to have seen more than once how a new subject can first be met with polite indifference, even skepticism, yet later be embraced as essential. That was the case with convex analysis. Now variational analysis is coming out of the shadows into brighter light as well. In my travels to regions around China, for instance, I find it being taught in surprisingly many academic circumstances, and elsewhere in the world, too. That's the wonderful story of mathematics. It can be very pure, simply motivated by curiosity perhaps, but if ultimately inspired by a true need, its day will come.

It's curious, by the way, to see notation and terminology of mine, even originally from my doctoral dissertation, now as ordinary as air in writings of others. It's deeply satisfying to think that my 66 years hard work were not only fun (for a mathematician), but may also have lasting impact.

This collection of papers marking my 90th birthday goes a long way toward reinforcing that feeling of satisfaction!

I'm amazed and utterly thankful to the many, many authors who have contributed. And so grateful to Lionel Thibault as editor, who spent endless hours in bringing this to fruition. And to all the reviewers who came through with the expert help that was needed. But this is an opportunity also to thank Norbert Heldermann who, as publisher of this journal, has been a bedrock of support ever since its founding, and has, that way, had an unforgettable impact on convex analysis and all of us who love it and its progeny.

Terry Rockafellar
Seattle, November 2024