

## In Memory of Hedy Attouch: Advances and Legacies in Mathematical Analysis

**Giuseppe Buttazzo**

*Dip. di Matematica, Università di Pisa, Italy  
buttazzo@dm.unipi.it*

**Michel Théra**

*Dép. de Mathématiques, Université de Limoges, France  
michel.thera@unilim.fr*

**Lionel Thibault**

*Dép. de Mathématiques, Université de Montpellier, France  
jca@umontpellier.fr*

In the fall of 2023, the mathematics community lost not only a prolific researcher, but also a valued collaborator and a remarkably kind mentor when Hedy Attouch passed away, on October 14, 2023, at the age of 74. As we edit this special issue in his honor, we would like to reflect on Hedy's life and remember his many achievements, his generous service to our community, and his deep and lasting contributions to Mathematical Analysis: Calculus of Variations, Convex Analysis, Dynamical Systems, Optimization, Partial Differential Equations, Variational Analysis, etc.

Hedy Attouch was born on February 16, 1949, in Toulouse (France). After completing his studies at the "Classes Préparatoires" at the Lycée Louis Le Grand à Paris and at the École Normale Supérieure de Cachan, Hedy's academic career began in 1971, when he obtained a position as Assistant Professor at the Université de Paris-Sud in Orsay. In 1976, he defended his *Thèse de Doctorat d'État* at the Université Paris VI under the supervision of Haïm Brezis. Afterward, he took a sabbatical in the United States and visited the Courant Institute, the University of Wisconsin, the University of Chicago, the University of Kentucky and the University of Washington, where he had the opportunity to meet several renowned mathematicians such as Louis Nirenberg, Michael Crandall, Felix Browder, Roger Wets, and Terry Rockafellar, respectively.

His encounter with Roger Wets marked the beginning of an active and fruitful collaboration spanning diverse interconnected areas, including epigraphic analysis, quantitative stability of variational systems, variational convergence of functions, etc.. This collaboration gave rise, in particular, to what is nowadays called the Attouch-Wets topology.

From Orsay, he moved to the Université de Perpignan in 1983 as a Professor, where he founded the AVAMAC laboratory (Variational Analysis and Applications to Mechanics, Automatics, and Control).

Then, in 1988, he joined the Université de Montpellier, more specifically the *Laboratoire d'Analyse Convexe*, where he was warmly welcomed by Jean Jacques Moreau –

one of the founders of modern convex analysis alongside Werner Fenchel and Terry Rockafellar – as well as by Charles Castaing and Michel Valadier, both renowned for their contributions to Convex Analysis, Set-Valued Analysis, and Young Measures.

It was in this laboratory that Hedy spent the rest of his career until his retirement. During his years in Montpellier, he trained many students, authored numerous papers, and played a key role in transforming the *Travaux du Séminaire d'Analyse Convexe de Montpellier* into a leading international journal, the *Journal of Convex Analysis*.

Hedy Attouch was a mathematician who worked at the interface between theoretical and applied mathematics. His research activity can be viewed as mainly spanning three distinct thematic periods:

- Variational convergence and maximally monotone operators (1972–1996)
- Continuous and discrete approaches in optimization (1997–2008)
- Development of fast algorithmic methods for convex and non-convex optimization problems (2009–2023)

During the first period, he was among the first to recognize that the class of maximally monotone operators – central, through the subclass of subdifferentials, to the development of the analysis of convex variational problems – are also fundamental in optimization in the large, partial differential equations, theory of semigroups, mechanics, etc. It was in his 1976 Thèse de Doctorat d'État that he provided one of his best-known results, named "Attouch Theorem" in the literature. This theorem, making a bridge between Convex Analysis and diverse Partial Differential Equations, relies on the typical instrumental convergence of extended real-valued convex functions and the convergence of their subdifferentials.

In the second period, by viewing most numerical algorithms as discrete dynamical systems and applying Lyapunov stability analysis for dissipative systems, he contributed to develop a deep and unified understanding of many optimization methods, particularly gradient-based and proximal algorithms. The visibility and impact of Hedy's research during the third period were truly remarkable. This was largely due to the relevance of his work across numerous applied fields in which optimization plays a central role, including computer science, engineering, statistics, and data science. He placed the powerful interplay between continuous dynamical systems and algorithms – often obtained through explicit or implicit time discretizations – at the heart of his research activity.

In recognition of the exceptional impact of his many outstanding contributions, Hedy received the prestigious George B. Dantzig Prize in 2021, awarded every three years jointly by the Mathematical Optimization Society (MOS) and the Society for Industrial and Applied Mathematics (SIAM). This honor recognized his fundamental contributions to modern variational analysis and non-continuous optimization, including the development of new notions of variational convergence, the introduction of novel topologies for the study of stability, the quantitative analysis of variational systems, and their applications in the design and analysis of algorithms, dynamical systems, and partial differential equations.

For more details on Hedy's research, the reader may consult the publication *Hedy Attouch, in memoriam* [Eur. Math. Soc. Mag. 132 (2024) 43–45].

Hedy Attouch wrote several books, including *Variational Convergence for Functions and Operators*, and *Variational Analysis in Sobolev and BV Spaces: Applications to PDEs and Optimization* co-authored with Giuseppe Buttazzo and Gérard Michaille. Both books are reference treatises on variational analysis, optimization, and PDEs, widely used by researchers and students in these fields.

Hedy also leaves behind an important scientific legacy through more than 170 outstanding publications and through the many doctoral students he supervised throughout his career. He trained around fifteen doctoral students, most of whom obtained academic positions; several of them went on to receive distinguished recognition, including prestigious awards.

Finally, it is impossible to speak of Hedy without recalling his kindness, his openness, and the generosity he showed to colleagues. He possessed a rare gift for communication and for sharing knowledge. His enthusiastic engagement with the international scientific community greatly enriched the field, and he leaves behind a substantial body of mathematical work upon which his many students and collaborators will continue to build.

Hedy was also an editor of the *Journal of Convex Analysis*. This memorial issue of the journal brings together contributions from about a hundred authors, covering the broad area of Hedy's research. We warmly thank all the authors and all the referees for their participation.

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Giuseppe Buttazzo, Michel Théra, Lionel Thibault